Asset Prices and Inequality in a Heterogenous OLG model

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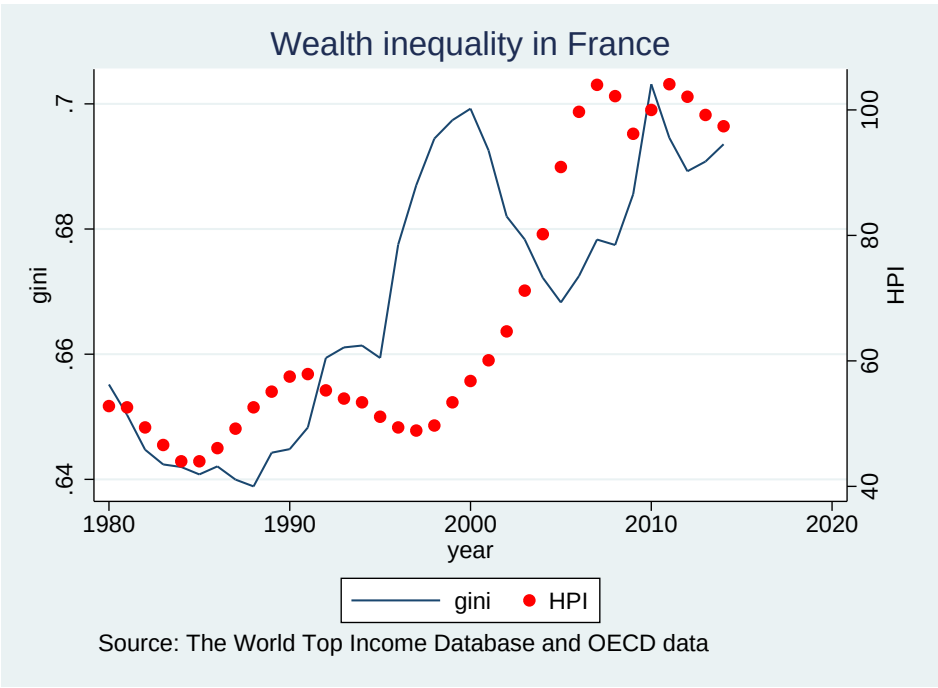
Introduction

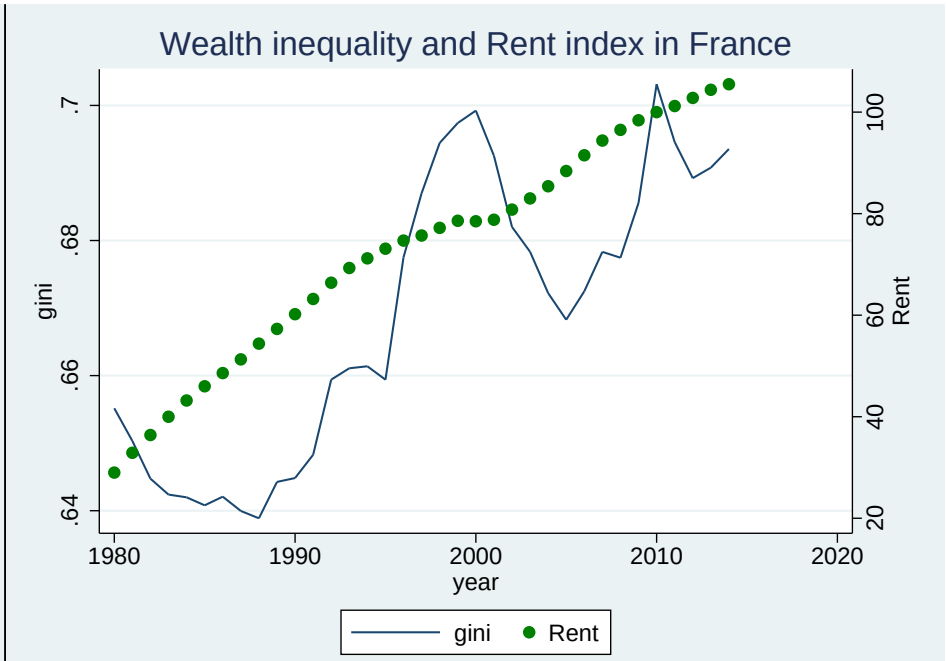
Wealth inequality is observed to constantly increase across many countries recent decades (Atkinson et al (2011), Saez and Zucman (2016))

Global house prices (in real terms) have also been on an upward trajectory. In the book of Thomas Piketty, the increase in wealth inequality is mainly driven by the rise of housing capital via price in many countries.

$\rightarrow$ *Are the two trends just a coincidence?*

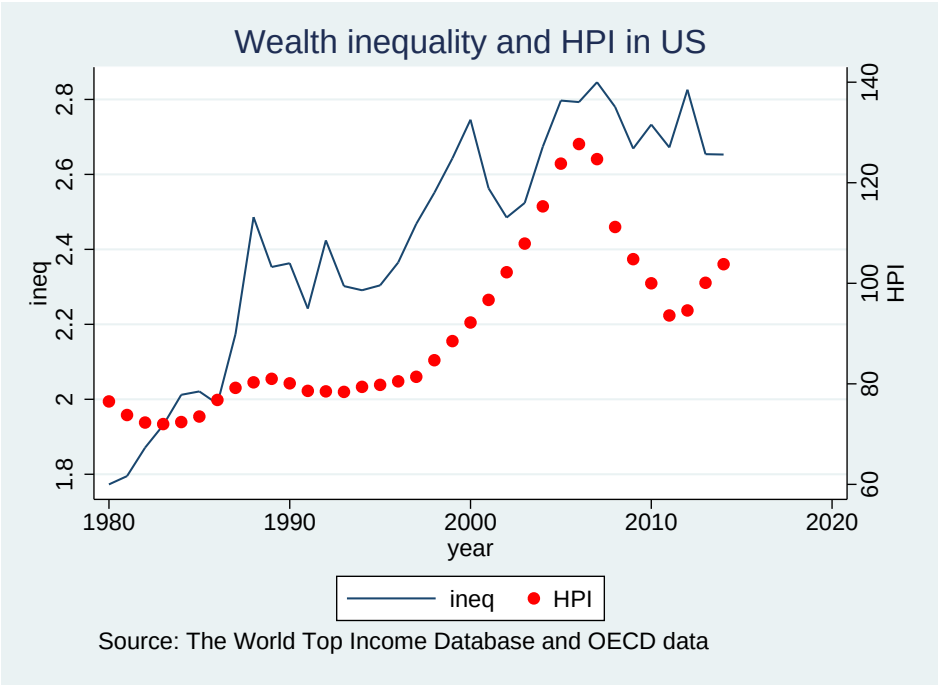


Source: The World Top Income Database and OECD data



Correlation= 0.58***

Correlation= 0.82***



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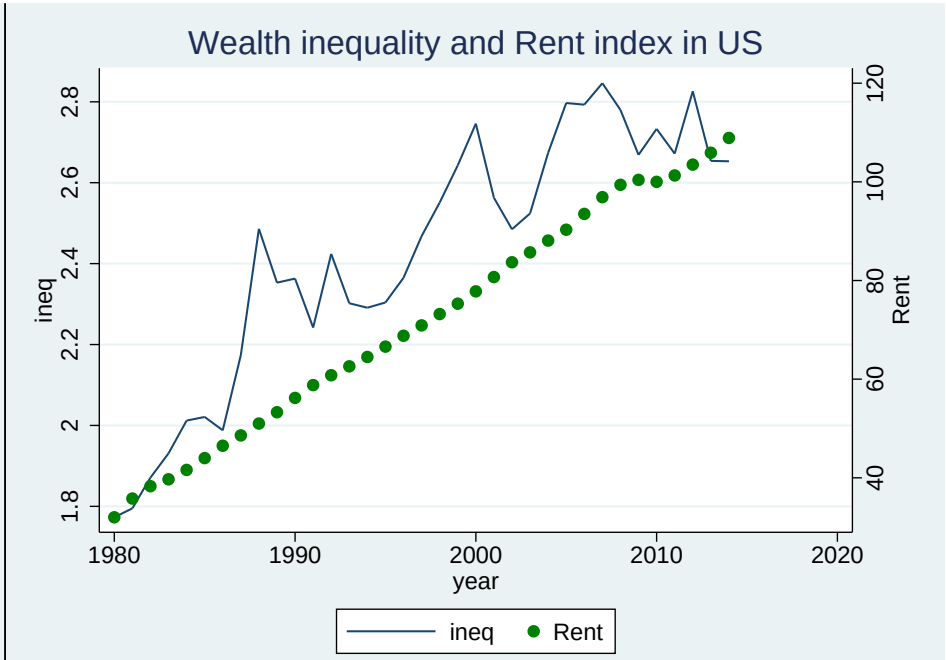

Correlation= 0.8^{***}

Correlation= 0.91^{***}

The link between Rents and wealth inequality is also positive.

With approximately 40% the population who rent (US, FR cases), income from rent should matter for the dynamics of wealth inequality via portfolio saving decision.

$\rightarrow$ *How housing price and rents involve in determining the distribution of wealth across HHs in the long-run?*

The relationship between homeownership rate and wealth inequality is recently depicted in the Household Finance and Consumption Survey (2013)

Inequality is lower in countries with higher homeownership rates (Bezrukovs (2013) and Kaas et al (2016)).

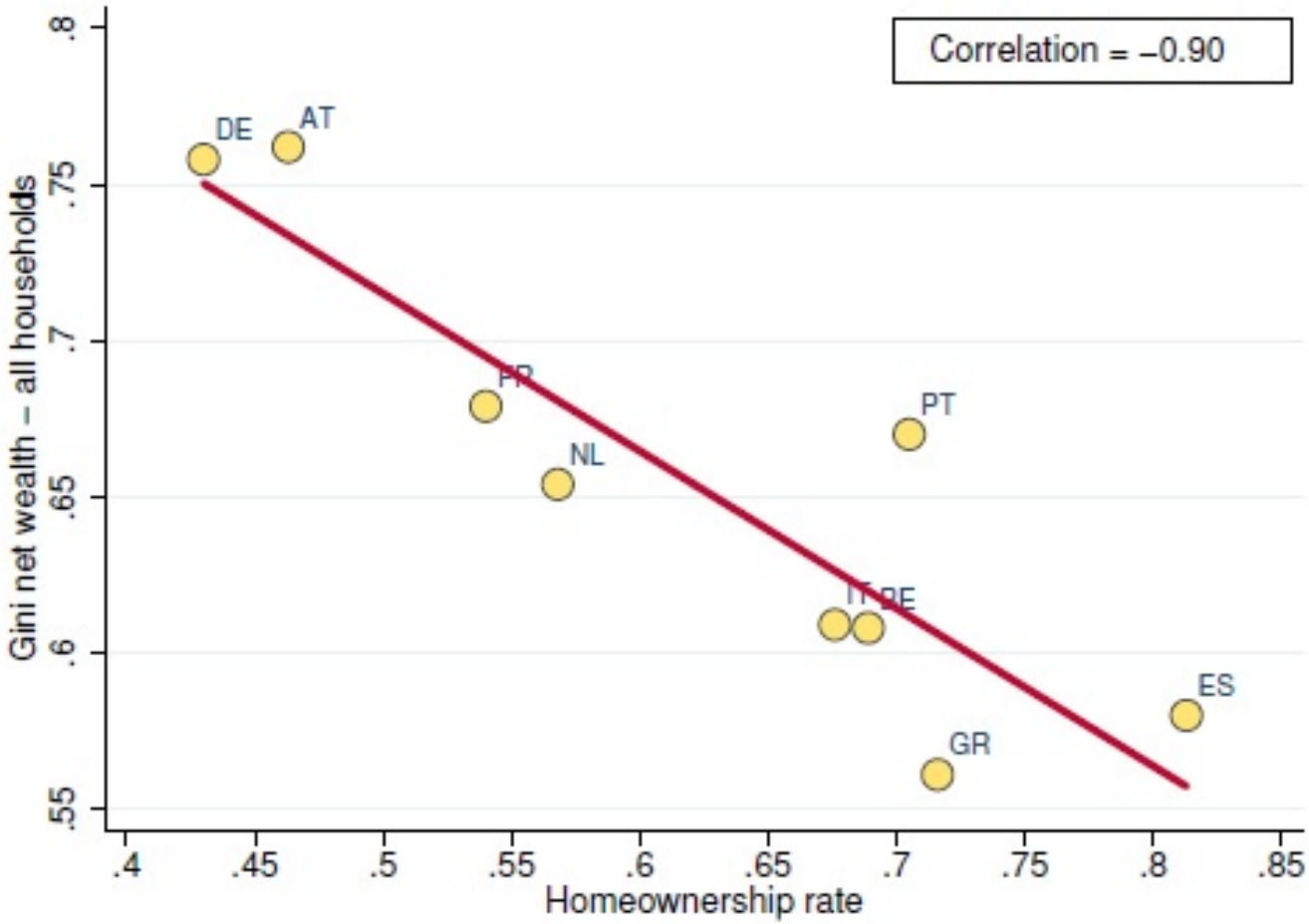

Figure 1: Wealth inequality and homeownership

Figure: Source:Household Finance and Consumption Survey (2013)

Related Literature

Literature studying housing with incomplete market models and heterogenous agents has been growing recently (Kiyotaki et al (2011),Sommer et al (2013), Iacoveillo and Pavan(2013)).

A link between housing price and inequality has been directly addressed in several papers.(Nakajima(2005), Ortalo-Magn and Rady (2006), Piazzesi and Schneider (2009))

Common literature often ignore the rental market. While it is true that renters are a minority in the total population, they represent an overwhelming majority among the poorer and younger age groups.

Matsuyama (2000) addressed the link between credit market and wealth inequality; however, the paper focuses on financial assets, which are different with housing asset.

Goal and Expectations

In the same spirit with Matsuyama (2000), but we focus on the connection between returns of housing market and wealth inequality.

Bequest is the only source of heterogeneity within households since it plays an important role in shaping wealth inequality. (Kotlikoff and Summers (1981), Gokhale et al (2001) , Piketty and Zucman(2014))

OLG models can take into account intergenerational transmission of wealth

Heterogenous individuals to replicate the economy: home-owners and renters

Prices, Rent, Interest and Homeownership rate are determined endogenously.

The model structure

Environment

Time is discrete and runs to infinity.

There are two types of households in the society: high endowment receiver (H) and low endowment one (L), i.e. they differ in the amount they are bequeathed.

Each household lives for 2 periods and there's no population growth.

There's no uncertainty

There're inter-vivo transfers that provide "warm glow".

There are two types of assets: housing and capital.

They may invest their savings in the competitive credit market (become a lender), which earns a gross return, or

They may buy houses with a mortgage (become a homeowner).

Mortgages are available, but home-buyers must satisfy a minimum down payment requirement.

Moreover, housing pays rent as an income but can only held at the sizes bigger than or equal to a minimum threshold.

Housing stock is fixed at H and never deteriorates.

Individual's plan

The optimization problem of a household of type i starting at any given time t where $i \in (H, L)$

$$Max \;\; V_t^i = (c_t^{i,1-\gamma} h_{R,t}^{i,\gamma})^{1-\beta} (d_{t+1}^{i,1-\sigma} b_{t+1}^{i,\sigma})^{\beta}$$

s.t.

$$c_t^i + s_t^i + r_t h_{R,t}^i + p_t h_{H,t}^i \leq w(1-\tau) + b_t^i + r_t h_{H,t}^i \tag{1}$$

$$d_{t+1}^i + b_{t+1}^i \leq w\tau + p_{t+1} h_{H,t}^i + s_t^i R_{t+1} \tag{2}$$

$$h_{H,t}^i \in (0, [\underline{h}, +\infty]) \tag{3}$$

$$s_t^i \geq -(1-\theta) p_t h_{H,t}^i \tag{4}$$

Remark:

$b_0^H > b_0^L$

b_0^H and b_0^L are given at period 0.

If $h_{H,t} < h_{R,t}$, he's called Renter; otherwise, if $h_{H,t} \geq h_{R,t}$ he's called Home-owner.

Inequality index

Define $\omega_t^i = w(1-\tau) + b_t^i$
The wealth inequality can be measured by:

$$\frac{\omega_{t+1}^H}{\omega_{t+1}^L} = \frac{h_{R,t}^H}{H - h_{R,t}^H} \frac{\frac{\beta}{(1-\beta)\gamma}(H - h_{R,t}^H) - (1-\theta)\frac{p_t}{r_t}}{1 + \frac{\beta}{(1-\beta)\gamma} h_{R,t}^H - \theta\frac{p_t}{r_t}}$$

where

$$\frac{p_t}{r_t} = \frac{\frac{H - h_{R,t}^H}{\gamma H}\frac{\omega_t^H}{\omega_t^L} - \frac{h_{R,t}^H}{\gamma H} + 1}{\theta - (1-\theta)\frac{\omega_t^H}{\omega_t^L}}$$

The degree of inequality depends on how prices p and rent r evolve.
$\frac{h_{R,t}^H}{H}$ captures the homeownership rate over time.

Wealth equality in equilibrium

When the constraint (3) and (4) don't bind for both types of households

We found the no-arbitrage condition:

$$R_{t+1} = \frac{p_{t+1}}{p_t - r_t} \tag{5}$$

Households are indifferent between housing and capital.

Wealth equality in equilibrium

Define the threshold $\underline{\omega_t^e} = \frac{(\theta p_t - r_t)\underline{h} + (1-\beta) w\tau / R_{t+1}}{\beta}$

Proposition

If individual's wealth $\omega_t^i > \underline{\omega_t^e}$, he will invest in housing.
Otherwise, he will save in the form of capital.

Proposition

In the equilibrium with equality: $R = \frac{p}{p-r}$ and all agents are indifferent between housing and capital.
Inequality vanishes in the long-run. Therefore, the long-run wealths of two types of households are equal to

$$\omega = \frac{w(1-\tau) + \sigma\beta\tau w}{1 - \sigma\beta R}$$

where

$$R = \frac{\gamma(1-\beta) + \beta}{\beta}$$

Wealth inequality in equilibrium

If only the constraint (4) binds, i.e. $s_t = -(1-\theta)p_t h_{H,t}$, there's an arbitrage opportunity for housing:

$$\frac{p_{t+1}}{p_t - r_t} > R_{t+1} \tag{6}$$

Therefore, all households want to borrow and invest as much as possible; however, only the one that is wealthy enough is able to borrow.

The wealth dynamics of the L-type (renter) and H-type (home-owner) is as follows:

$$\omega_t^H = \sigma\beta\rho_t(\omega_{t-1}^H + \frac{w\tau}{\rho_t}) \tag{7}$$

$$\omega_t^L = \sigma\beta R_t(\omega_{t-1}^L + \frac{w\tau}{R_t}) \tag{8}$$

where the discount factor of the H-type is

$$\rho_t = \frac{p_t - (1-\theta)R_t p_{t-1}}{\theta p_{t-1} - r_{t-1}}$$

$$\text{and } \rho_t > R_t$$

Define the wealth threshold:

$$\underline{\omega_t^i} = \frac{(\theta p_t - r_t)\underline{h} + (1-\beta)\frac{w\tau}{\rho_{t+1}}}{\beta}$$
$$= \frac{\theta}{\beta}(\underline{h} - (1-\beta)H)p_t + (\frac{1-\beta}{\beta} + \frac{h_{R,t}^H}{\gamma} - \underline{h})r_t \quad (9)$$

Proposition

If $\omega_t^i < \underline{\omega_t^i}$, the type i individual won't possess any houses. He will become a renter.
On the other hand, if $\omega_{i,t} > \underline{\omega_t^i}$, the type i agent will buy all houses available and become home-owner.
There will be no equilibrium in which the two individuals are home buyer.

Proposition

In the equilibrium with inequality: $R < \frac{p}{p-r}$, *the long-run wealth of the type H will be* $\omega^H = \frac{w(1-\tau)\rho+\sigma\beta\tau w}{1-\sigma\beta\rho}$ *while the one of type L will be* $\omega^L = \frac{w(1-\tau)R+\sigma\beta\tau w}{1-\sigma\beta R}$. *Of course,* $\omega^H > \omega^L$.

The steady states $(\omega^H; \omega^L)$ *are unique.*

Comparative Statistics

Table: Impact from a change in parameter τ and θ
τ: contribution rate and θ: down-payment rate

	$\downarrow \theta$	$\uparrow \tau$
ω^H/ω^R	$\downarrow$	$\downarrow$
h_R^H	$\uparrow$	$\uparrow$
p	$\uparrow$	$\downarrow$
r	$\downarrow$	$\downarrow$
$\frac{p}{r}$	$\uparrow$	$\downarrow$

Long-run relationship between inequality and homeownership rate

When the borrowing constraint is loose, i.e. θ is lower, wealth inequality decreases while the fraction of homes occupied by owners increases.

Long-run relationship between inequality and house price-rent ratio

A change in the borrowing constraint lead to an opposite adjustment between wealth inequality and price-to-rent ratio in the long-run.

Pension reform and wealth inequality

Consider a higher contribution rate to the pension system: $\partial\tau > 0$.

A pension reform will have a direct income effect, and

it also has indirect effect via the housing market

Pension reform leads to inequality reduction

A reform that increases the contribution rate in the pension system leads to a lower inequality via an reduction in price-rent ratio. In this case, the negative impact on the type H agent is larger than the one that affected the type L individual.

Long-run relationship between inequality and homeownership rate

When the contribution rate is higher, i.e. τ is higher, wealth inequality decreases while the fraction of homes occupied by owners increases.

Main findings

- The model explains how housing wealth and wealth inequality will interact when there's a shock on downpayment rate or when there's pension reform.
- The model explains why wealth inequality is lower in countries/places where homeownership rate is higher.
- For further research: Introduce the fiscal policy to reduce inequality.
- Endogenize borrowing constraint